

Effect of random synaptic dilution on recalling dynamics in an oscillator neural network

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In the present paper, we study the effect of random synaptic dilution in an oscillator neural network in which information is encoded by the relative timing of neuronal firing. In order to analyze the recalling process in this oscillator network, we apply the method of statistical neurodynamics. The results show that the dynamical equations are described by some macroscopic order parameters, such as that representing the overlap with the retrieved pattern. We also present the phase diagram showing both the basin of attraction and the equilibrium overlap in the retrieval state. Our results are supported by numerical simulation. Consequently, it is found that both the attractor and the basin are preserved even though dilution is promoted. Moreover, as compared with the basin of attraction in the traditional binary model, it is suggested that the oscillator model is more robust against the synaptic dilution. Taking into account the fact that oscillator networks contain more detailed information than binary networks, the obtained results constitute significant support for the plausibility of temporal coding. [S1063-651X(98)11405-8]

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I. INTRODUCTION

In recent years, many attempts have been made to treat neural network models more realistically than did traditional ones such as the Hopfield model [1]. Progress in the physiological understanding of real neural systems, for example, neuronal activity and morphology of synaptic connectivities, has led theoretical interests to those various models with biological validity. With this trend, many models whose purpose is to grasp the essence of more detailed dynamics in a neuron have been proposed and analyzed. From the theoretical point of view, models capable of describing the continuous behavior of neuronal activities are expected to be superior in information processing. In particular, oscillator neural networks have come to be one of the most intriguing models in this context, since it was reported that collective oscillatory behavior may contribute to information processing in biological systems [2,3]. This is because such models are simple enough to allow for theoretical analysis, while they also contain the essence of the temporal features of neuronal activity. The results obtained through the analysis of such a simple model are sure to further our understanding not only of more complex models but of real neural systems.

Many interesting analyses concerning oscillator neural networks have been reported [4–11]. However, we will only briefly survey the results related to our model. In the case of autoassociation of random phase patterns with Hebbian learning, the storage capacity $\alpha_c=0.038$ is estimated using the replica theory [12,13]. An oscillator model with sparse connectivity has been analyzed by Noest [14]. We have investigated equilibrium properties in the situation involving synaptic damage [15]. Recently, we have studied the retrieval dynamics for both autoassociation and phase sequence generator [16]. However, despite these works, oscillator models remain poorly understood.

For the purpose of estimation of performance with regard to associative memory, it is necessary to consider two important aspects of association, one concerning the attractor and the other concerning the basin. However, most previous

studies are restricted to properties of the attractor, such as equilibrium overlap and storage capacity. However, considering the associative ability for a noisy pattern to be dynamically corrected, in order to obtain a proper measure of a model's performance, it is necessary to study the basin of attraction also. In order to discuss the basin, we must treat the dynamics of recalling.

For traditional models, several theoretical analyses on dynamics of retrieval processes have been reported. Using the method of generating functionals and path integrals, a general theory can be formulated [17]. Although this method yields an exact description, a suitable approximation is required in order to obtain practical results [18]. In the case of parallel dynamics, the result turns out to be simple so that the exact prediction of the retrieval dynamics for the initial few steps is possible [19]. As for arbitrary finite time scale, the dynamical replica theory has been proposed recently [20]. On the other hand, as an approximation method, the statistical neurodynamics theory has been proposed [21,22]. Although the approximation used in this approach is crude in a sense, it is practically useful to predict long term behavior when a network succeeds in retrieval [23]. Finally, we should note that, under suitable conditions, the theoretical result from the statistical neurodynamics can be obtained also by the path integral method.

In the present paper, we discuss the retrieval dynamics in an oscillator network with diluted synapses. However, it is expected that the rigorous treatment for such a network is more complicated than that for traditional binary models. Therefore, following our previous work [16], we apply the theory of statistical neurodynamics, which enable us to obtain fruitful results more easily. In the next section, we introduce an oscillator neural network model treated here. Section III contain a theoretical analysis of the recalling process in this network. Using the derived dynamical equations describing the time development of some macroscopic parameters such as that representing overlap, we examine the effect of dilution on both the attractor and the basin. These theoretical results are verified by means of numerical simulation

in Sec. IV. In Sec. V, we give a brief summary and conclusion.

II. OSCILLATOR NEURAL NETWORK MODEL

We now describe the model treated in the present paper. The state of the i th neuron at time t is represented by

$$S_i(t) = \exp[i\phi_i(t)], \quad (1)$$

where $\phi_i(t)$ is the phase of the i th neuron at time t . We should remark that this phase corresponds to the timing of neuronal firing. The oscillator network evolves according to the synchronous dynamics:

$$S_i(t+1) = \frac{h_i(t)}{|h_i(t)|}, \quad h_i(t) = \sum_{j \neq i}^N J_{ij} S_j(t), \quad (2)$$

where $h_i(t)$ is the internal potential of the i th neuron at time t and J_{ij} is the synaptic connection between the i th and the j th neurons.

We construct this model not only as a simple extension of traditional models but following a theoretical basis of coupled oscillator systems. The system consists of a number of nonlinear oscillatory units. Although, in general, even the behavior of such a unit is described by a dynamical equation involving a set of state variables, it is well known that such a system can be reduced to a system characterized by simple phase variables under suitable conditions [24]. The equations obtained with such a reduction technique are given by

$$\frac{d\phi_i}{dt} = \sum_{j \neq i}^N K_{ij} \sin(\phi_j - \phi_i + \beta_{ij}). \quad (3)$$

To interpret the above system in the context of traditional neural networks, we introduce the complex representation $S_i = \exp(i\phi_i)$ into the above equation. Then, the equilibrium state satisfies the conditions

$$S_i = \frac{h_i}{|h_i|}, \quad h_i = \sum_j^N J_{ij} S_j, \quad (4)$$

where $J_{ij} = K_{ij} \exp(i\beta_{ij})$. Clearly, the equilibrium state (4) of the dynamics described by Eq. (3) is identical with that of the dynamics described by Eq. (2). However, generally, owing to different dynamics, these systems may exhibit quantitative or even qualitative differences. Numerical simulation suggests that these differences are in most cases negligible [16].

The synaptic connection J_{ij} should be determined appropriately so that the system can perform as an associative memory model. We denote the set of patterns to be memorized by the network as complex variables, $\xi_i^\mu = e^{i\theta_i^\mu}$. The value ξ_i^μ represents the state of the i th unit in the μ th pattern. Here, we examine the case of random patterns; θ_i^μ is a uniform random number between 0 and 2π . To make the network memorize p such phase patterns, we construct synaptic connections J_{ij} by applying the generalized Hebbian rule:

$$J_{ij} = \frac{1}{N} \sum_{\mu=1}^p \xi_i^\mu \bar{\xi}_j^\mu, \quad (5)$$

where $\bar{\xi}_j^\mu$ is the complex conjugate of ξ_j^μ .

To discuss robustness against damage of synaptic connections, we define

$$\bar{J}_{ij} = \frac{c_{ij}}{c} J_{ij}, \quad (6)$$

as the formulation for randomly diluted synapses. Here, the c_{ij} are independent random variables, which assume the values 1 and 0 with probabilities c and $1-c$, respectively. Note that the dilution parameter c represents the ratio of connections. In the limit $N \rightarrow \infty$, the expression in Eq. (6) can be regarded as that of synaptic connections with static noise [25],

$$\bar{J}_{ij} = J_{ij} + \eta_{ij}. \quad (7)$$

The synaptic noise η_{ij} is a complex Gaussian noise with mean 0 and variance η^2/N . It is easy to determine the relation between the dilution ratio c and the variance parameter η^2 as

$$\eta^2 = \frac{1-c}{c} \alpha. \quad (8)$$

For the sake of simplicity in later theoretical analysis, we adopt the expression of Eq. (7).

To investigate various features of the recalling process, it is useful to define the overlap

$$M^\mu(t) = m^\mu(t) e^{i\varphi^\mu(t)} = \frac{1}{N} \sum_{j=1}^N \bar{\xi}_j^\mu S_j(t) \quad (9)$$

as the order parameter of our system. As the network's configuration $S_i(t)$ approaches the target pattern ξ_i^μ , the absolute value m_μ increases to unity.

Finally, in the retrieval process of the oscillator neural network

$$S_i(t+1) = \frac{h_i(t)}{|h_i(t)|}, \quad (10)$$

$$h_i(t) = \sum_{j \neq i}^N \bar{J}_{ij} S_j(t), \quad (11)$$

$$\bar{J}_{ij} = \frac{c_{ij}}{Nc} \sum_{\mu=1}^p \xi_i^\mu \bar{\xi}_j^\mu, \quad (12)$$

we mainly discuss the time development of the overlap (9) along with certain other macroscopic parameters.

III. THEORETICAL ANALYSIS

Let us consider the situation in which the network is recalling the pattern ξ_i^1 , namely,

$$m^1(t) \sim O(1), \quad m^\mu(t) \sim O\left(\frac{1}{\sqrt{N}}\right) \quad (\mu \neq 1). \quad (13)$$

The internal potential $h_i(t)$ in Eq. (2) can be separated as

$$\sum_{j \neq i}^N \bar{J}_{ij} S_j(t) = \xi_i M(t) + \frac{1}{N} \sum_{j \neq i}^N \sum_{\mu=2}^p \xi_i^\mu \bar{\xi}_j^\mu S_j(t) + \sum_{j \neq i}^N \eta_{ij} S_j(t), \quad (14)$$

where $\xi_i = \xi_i^1$ and $M(t) = M^1(t)$. From this point, for simplicity, we drop the index μ in the case of pattern 1. In this process, the first term on the right hand side (rhs) of Eq. (14) is regarded as the signal to induce recollection of the target pattern ξ_i^1 , while the remaining terms are regarded as noise. For convenience, we define the noise terms $z_i(t)$ as

$$z_i(t) = z_i^c(t) + z_i^s(t) = \frac{1}{N} \sum_{j \neq i}^N \sum_{\mu=2}^p \xi_i^\mu \bar{\xi}_j^\mu S_j(t) + \sum_{j \neq i}^N \eta_{ij} S_j(t). \quad (15)$$

In $z_i(t)$, $z_i^c(t)$ is the crosstalk noise from unretrieved patterns ($\mu \neq 1$), and $z_i^s(t)$ is caused by noise in the synapses. The essence of the theory is to treat the crosstalk noise $z_i^c(t)$ as complex Gaussian noise with mean 0 and variance $\sigma_c(t)^2$. It has been confirmed numerically that this assumption is valid as long as the network succeeds in retrieval [23]. In addition, the synaptic noise $z_i^s(t)$ is also assumed to be complex Gaussian with mean 0 and variance $\sigma_s(t)^2$ [25]. Therefore, $z_i(t)$ displays a complex Gaussian distribution with mean 0 and variance $2\sigma_t^2 = \sigma_c(t)^2 + \sigma_s(t)^2$. Here, we also assume $z_i^c(t)$ and $z_i^s(t)$ to be independent. We note that $z_i(t)$ can be expressed with two independent Gaussian variables $x_i(t)$ and $y_i(t)$ satisfying

$$z_i(t) = x_i(t) + iy_i(t), \quad x_i(t), y_i(t) \sim N(0, \sigma(t)^2), \\ \langle x_i(t) y_i(t) \rangle = 0. \quad (16)$$

Now we derive a dynamical equation for the overlap with the recalled pattern. The definition of overlap (9) leads to the equation

$$m(t+1) e^{i\varphi(t+1)} = \frac{1}{N} \sum_j^N \frac{\xi_j m(t) e^{i\varphi(t)} + z_j(t)}{|\xi_j m(t) e^{i\varphi(t)} + z_j(t)|}. \quad (17)$$

The variable $z_j(t)$ represents Gaussian noise. Then, because of the symmetry of its distribution, we assume $z_j(t)$ produces no effect to change $\varphi(t)$. This assumption has been confirmed numerically. Using this assumption, i.e., setting $\varphi(t) = \varphi_0$, we obtain

$$m(t+1) = \frac{1}{N} \sum_{j=1}^N \frac{m(t) + z_j(t) e^{-i(\varphi_0 + \theta_j)}}{|m(t) + z_j(t) e^{-i(\varphi_0 + \theta_j)}|} \\ = \frac{1}{N} \sum_{j=1}^N \frac{m(t) + z_j(t)}{|m(t) + z_j(t)|}. \quad (18)$$

Here, we use the fact that the distribution of $z_j(t) e^{-i(\varphi_0 + \theta_j)}$ can be obtained by simply rotating that of $z_j(t)$.

Next, we examine the time development of the variance $\langle |z_i(t)|^2 \rangle = 2\sigma(t)^2$. First, we consider the synaptic noise $z_i^s(t+1) = \sum_{j \neq i}^N \eta_{ij} S_j(t+1)$. When we take the statistics of $z_i^s(t+1)$, we must take into account correlations between η_{ij} and η_{ji} in $S_j(t+1)$. Here, expanding $S_j(t+1) = h_j(t)/|h_j(t)|$ in terms of η_{ji} yields

$$z_i^s(t+1) = \sum_{j \neq i}^N \eta_{ij} \frac{h_j^0(t)}{|h_j^0(t)|} + S_i(t) \sum_{j \neq i}^N \frac{\eta_{ij} \eta_{ji}}{2|h_j^0(t)|}, \quad (19)$$

where $h_j^0(t)$ is assumed to be independent of η_{ji} . If the dilution is asymmetric, $\eta_{ji} \neq \bar{\eta}_{ij}$ (or $c_{ij} \neq c_{ji}$), the second term vanishes. Even if it were symmetric, the assumption that the mean of the noise is 0 would lead us to neglect the second term proportional to $S_i(t)$, since it is related to the mean of $z_i^s(t+1)$. As a result, we obtain

$$\sigma_s(t+1)^2 = \eta^2. \quad (20)$$

Second, consider the crosstalk noise $z_i^c(t+1)$. We express $z_i^c(t+1)$ as

$$z_i^c(t+1) = \frac{1}{N} \sum_{j \neq i}^N \sum_{\mu=2}^p \xi_i^\mu \bar{\xi}_j^\mu \frac{h_j(t)}{|h_j(t)|}. \quad (21)$$

When summing over μ , as in the case of Eq. (19), we must consider the dependence of $h_j(t)$ on ξ_j^μ . In the local field $h_j(t)$, the term

$$\frac{1}{N} \sum_{k \neq j}^N \xi_j^\mu \bar{\xi}_k^\mu S_k(t) \sim \xi_j^\mu M^\mu(t), \quad (22)$$

which is caused by the non-target pattern μ , is estimated to be $O(1/\sqrt{N})$. Using this estimation, we expand the complex function $h_j(t)/|h_j(t)|$, obtaining

$$z_i^c(t+1) = \frac{1}{N} \sum_{j \neq i}^N \sum_{\mu=2}^p \xi_i^\mu \bar{\xi}_j^\mu \frac{h_j^\mu(t)}{|h_j^\mu(t)|} \\ + \frac{1}{N} \sum_{j \neq i}^N \frac{1}{2|h_j^\mu(t)|} \frac{1}{N} \sum_{k \neq i}^N \sum_{\mu=2}^p \xi_i^\mu \bar{\xi}_k^\mu S_k(t) \\ + O\left(\frac{1}{\sqrt{N}}\right), \quad (23)$$

where $h_j^\mu(t) = \xi_j M(t) + (1/N) \sum_{k \neq j}^N \sum_{\nu \neq 1, \mu}^p \xi_j^\nu \bar{\xi}_k^\nu S_k(t) + z_s(t)$ is assumed to be independent of ξ_j^μ . Accordingly, we find

$$z_i^c(t+1) = \frac{1}{N} \sum_{j \neq i}^N \sum_{\mu=2}^p \xi_i^\mu \bar{\xi}_j^\mu S_j(t+1) + U(t) z_i^c(t), \quad (24)$$

and

$$U(t) = \frac{1}{N} \sum_{j=1}^N \frac{1}{2|\xi_j M(t) + z_j(t)|}. \quad (25)$$

where we have used the fact $h_j^\mu(t) \rightarrow h_j(t)$ in the limit $N \rightarrow \infty$ in Eq. (24). Squaring Eq. (24) and averaging in order to obtain $\sigma_c(t+1)$, we obtain

$$\begin{aligned} \sigma_c(t+1)^2 &= \alpha + U(t)^2 \sigma_c(t)^2 \\ &+ 2U(t) \text{Re} \left\langle \frac{1}{N} \sum_{j \neq i}^N \sum_{\mu=2}^p \xi_i^\mu \bar{\xi}_j^\mu S_j(t+1) \bar{z}_i^c(t) \right\rangle, \end{aligned} \quad (26)$$

where $\alpha = p/N$.

We can calculate the last term in Eq. (26) by means of substituting Eq. (24) into Eq. (26) iteratively. Then, we need the following quantities:

$$\begin{aligned} X(t+1, t+1-\tau) &= \text{Re} \left[\frac{1}{N} \sum_j S_j(t+1) \bar{S}_j(t+1-\tau) \right] \\ &= \text{Re} \left[\frac{1}{N} \sum_j \frac{\xi_j M(t) + z_j(t)}{|\xi_j M(t) + z_j(t)|} \frac{\bar{\xi}_j \bar{M}(t-\tau) + \bar{z}_j(t-\tau)}{|\bar{\xi}_j \bar{M}(t-\tau) + \bar{z}_j(t-\tau)|} \right]. \end{aligned} \quad (27)$$

To carry out the average in the above equation, we must generally take account of the correlation $2C(t, t-\tau) = \langle z(t) \bar{z}(t-\tau) \rangle$. The estimation so obtained up to the n th preceding time step is called the n th order approximation [22]. In the n th order approximation, we assume that the noise at each time, $z(t-1), \dots, z(t-n+1)$, is correlated to $z(t)$, while $z(t-n)$ is independent of $z(t)$. Using Eq. (24) as Eq. (26) was used above, we can obtain equations for $C(t, t-\tau)$.

Finally, the macrodynamical equations for the n th order approximation are given as follows:

$$m(t+1) = \left\langle \frac{m(t) + z(t)}{|m(t) + z(t)|} \right\rangle_{z(t)}, \quad (28)$$

$$U(t) = \left\langle \frac{1}{2|m(t) + z(t)|} \right\rangle_{z(t)}, \quad (29)$$

$$\begin{aligned} 2\sigma(t+1)^2 &= \alpha + 2U(t)^2 \sigma(t)^2 + 2\alpha \sum_{\tau=1}^n X(t+1, t+1-\tau) \\ &\times \prod_{k=1}^{\tau} U(t+1-k) + [1 - U(t)^2] \eta^2, \end{aligned} \quad (30)$$

where

$$\begin{aligned} X(t+1, t+1-\tau) &= \text{Re} \left\langle \frac{m(t) + z(t)}{|m(t) + z(t)|} \frac{m(t-\tau) + z(t-\tau)}{|m(t-\tau) + z(t-\tau)|} \right\rangle_{z(t), z(t-\tau)}, \end{aligned} \quad (31)$$

and

$$2C(t, t-\tau) = \begin{cases} \alpha X(t, t-\tau) + 2U(t-1)U(t-\tau-1)C(t-1, t-\tau-1) \\ + \alpha \sum_{k=\tau+1}^{n-1} X(t, t-k) \prod_{\kappa=\tau+1}^k U(t-\kappa) \\ + \alpha \sum_{k=1}^{n-1} X(t-k, t-\tau) \prod_{\kappa=1}^k U(t-\kappa) + \eta^2 X(t, t-\tau) & (1 \leq \tau \leq n-2) \\ \alpha X(t, t-\tau) + 2U(t-1)C(t-1, t-\tau) + \eta^2 X(t, t-\tau) & (\tau = n-1) \\ 0 & (\tau = n). \end{cases} \quad (32)$$

We have assumed here that the site average $(1/N) \sum_j \dots$ does not depend on the memorized pattern and, for a given $m(t)$, $\sigma(t)$, does not depend on the initial pattern. In this case, this average is identical to $\langle \dots \rangle_{z(t)}$, where $z(t)$ represents an arbitrary $z_i(t)$. Here, we should note that $U(t)$ is given by $m(t), \sigma(t)$, and $X(t+1, t+1-\tau)$ by $m(t), m(t-\tau), \sigma(t), \sigma(t-\tau)$. When calculating these coupled equations (28), (29), (30), (31), and (32), it is necessary to give initial conditions $m(0)$, $\sigma(0)^2 = \alpha/c$, and $X(t, 0) = m(t)m(0)$ as an approximation.

IV. THE RESULTS

First, we show the time evolution of the overlap for several initial overlaps. In Fig. 1, (a) and (b) display typical

behavior found through numerical simulations and in theoretical analyses, respectively. It is observed that the two sets of results are in good agreement, especially when the network succeeds in retrieval. In both figures, while the curves starting at the initial overlap $m(0) \geq 0.3$ reach the equilibrium value $m(\infty) \sim 1$, the curves for $m(0) = 0.1$ decrease. In this case, the basin is found to lie between 0.1 and 0.3.

Figure 2 indicates results of theoretical analysis and numerical simulation in the case of $\eta = 0$ (or $c = 1.0$). The upper part and the lower part of the theoretical curves represent the equilibrium overlap $m_\infty(\alpha)$ and the basin of attraction $m_0(\alpha)$, respectively. Both of these are obtained as functions of α . The vertical parts of the curves represent the storage capacity α_c . Of the four curves, the fourth order approximation is in best agreement with the numerical simu-

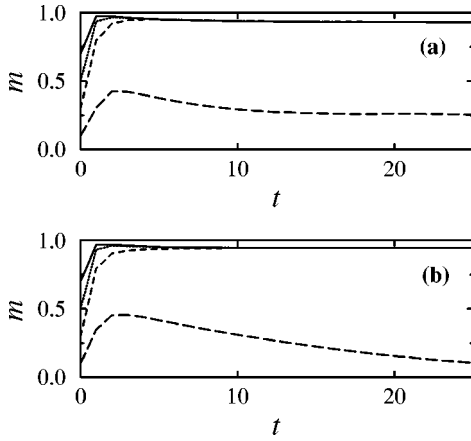


FIG. 1. Typical time evolution of overlaps for $\alpha=0.03$, $c=0.5$, and the initial overlaps $m=0.1, 0.3, 0.5$, and 0.7 . (a) Numerical simulation with $N=1000$. (b) Theoretical curves for the fourth-order approximation.

lations. From this result, it is found that we must take account of higher order temporal correlations of noise $z(t)$ to predict the behavior of the present model.

For various values of the dilution parameter c , numerical simulation and theoretical analysis were carried out. These results are given in Fig. 3. Here, the simulations were done in the case of symmetric dilution ($c_{ij}=c_{ji}$) and asymmetric dilution ($c_{ij}\neq c_{ji}$). We confirm that there is no discrepancy between in the two cases as long as we consider statistical properties. Moreover, it is observed that theoretical results are consistent with the simulations in this case as well.

In Fig. 4, we display the dependence on the ratio of connected synapses c . If $c\geq 0.3$, although the vertical lines move to left, the upper and lower curves are affected only slightly. The two curves do not approach each other until c reaches 0.1. In a previous study [15], we found that, for the case of symmetric dilution, equilibrium overlap remains comparatively large even if c is quite small. In the present

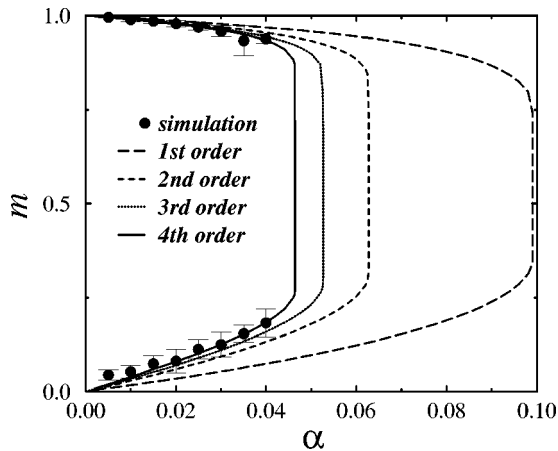


FIG. 2. The equilibrium overlap and basin of attraction. The four curves represent the theoretical results for various order approximations. The ordinate is the overlap m and the abscissa is the storage ratio α . The data points indicate simulation results with $N=1000$ for 20 trials. The upper part, the lower part, and the vertical part of the theoretical curves represent the equilibrium overlap, the basin of attraction, and the storage capacity, respectively.

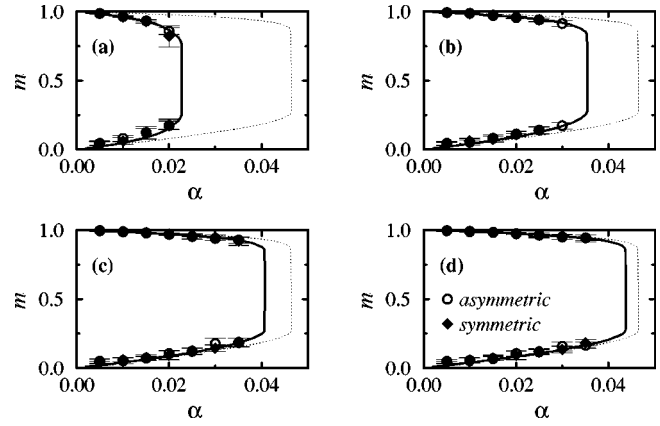


FIG. 3. Comparison between the effects of symmetric and asymmetric dilution. The solid curves represent the theoretical results at fourth order. The ratios of connected synapses are (a) $c=0.1$, (b) $c=0.3$, (c) $c=0.5$, and (d) $c=0.7$. For reference, the result for the case $c=1.0$ (i.e., the fully connected case) is indicated by the dashed curves.

study, this has been confirmed in the case of asymmetric dilution as well. Furthermore, we have found that the basin remains sufficiently wide even for small values of c .

We compare the width of the basin of the present model with that of the traditional model in Fig. 5. Since the network retrieves the target pattern when $m(0)>m_0(\alpha)$, we adopt $1-m_0(\alpha)$ as the width of the basin. This figure contains a plot of $1-m_0(0.8\alpha_c^{4th})$ for each model. Here, α_c^{4th} is the storage capacity obtained with the fourth order approximation. Making such a comparison, we see that the oscillator model has wider basin and is more robust against synaptic dilution.

V. CONCLUSION

Applying the theory of statistical neurodynamics to oscillator neural networks, we have obtained clear results for the case of random synaptic dilution. We found that numerical simulations support our theoretical results. The main results obtained in this study are as follows.

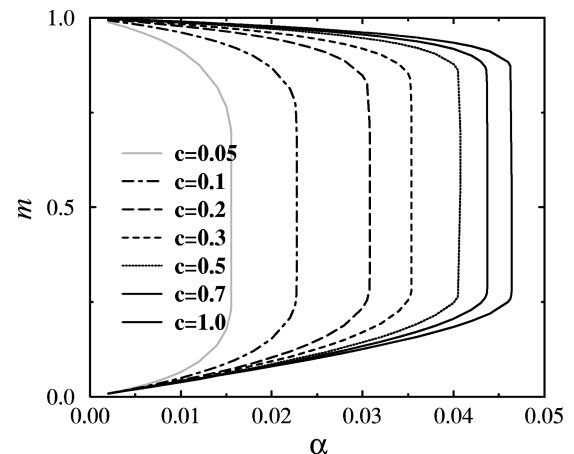


FIG. 4. Dependence of the theoretical curve on the ratio of connected synapses c . These curves were obtained with the fourth order approximation.

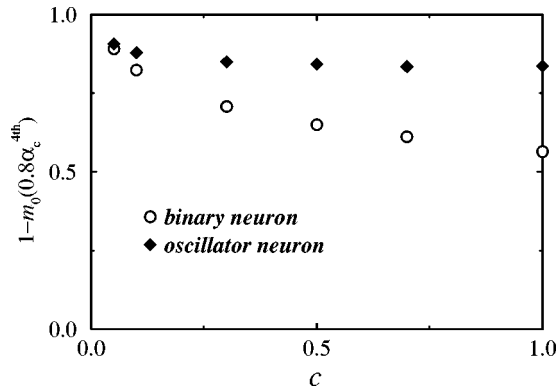


FIG. 5. Comparison of the sizes of basins for the oscillator and binary models near saturation, α_c^{4th} .

(i) In order to describe the properties of the networks theoretically, we must take into account the higher order temporal correlations of noise. The present study shows that it is necessary to consider at least the fourth order approximation.

(ii) For all values of c , theoretical results are in good agreement with numerical simulation. These theoretical curves indicate equilibrium overlaps and basins change little even if c decreases to about 0.3. Furthermore, the basins remain sufficiently wide near saturation.

(iii) The widths of basins in the oscillator model are wider than that in the binary model. Moreover, the oscillator model is found to be more robust against decrease of c .

In conclusion, we have found that the oscillator neural network exhibits good performance while processing detailed information such as the timing of neuronal firings. Our results support the plausibility of temporal coding.

Finally, let us make a comment leading to future problems. When we carried out numerical simulation of the oscillator neural networks, it was observed that a network is rarely trapped in spurious states. This may be one factor responsible for the ability of the oscillator network to recall from considerably noisy patterns. Though this point has not yet been investigated in the general case, the case of $\alpha=0$ (that in which the number of patterns p remains finite in the limit $N \rightarrow \infty$) has been investigated, and it has been reported that the symmetric mixture states are all unstable [11,26,27]. Investigation of the case for finite α should provide a deeper understanding of characteristics of oscillator neural networks.

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